BP307T · B.PHARM THEORY

BP307T Pharmaceutical Engineering

Unit V — Flow of Fluids

Pills & Python Notes

Types of flow · Manometers · Reynolds number · Bernoulli's theorem · Flow measurement devices

Unit V

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BP307T Pharmaceutical Engineering — Unit V: Flow of Fluids and Flow Measurement

Unit V (4 hours): Types and measurement of flow · Manometers · Reynolds number and its significance · Bernoulli's theorem and its applications · Orifice meter, Venturimeter, Pitot tube and Rotameter.
Chapter 3: Flow Basics. Chapter 4: Flow Measurement Devices.

CHAPTER 3 — FLOW BASICS: Types of Flow, Flow Measurement, Manometers, Reynolds Number, Bernoulli's Theorem

3.1 Learning objectives

After studying this chapter, the student will be able to:

- Define flow, flow rate, velocity, pressure, viscosity and density in plain words.
- Distinguish between laminar, turbulent and transitional flow, and between steady/unsteady, compressible/incompressible and viscous/inviscid flow.
- Calculate volumetric flow rate using Q = A × v, and mass flow rate from volumetric flow rate.
- Describe the construction, working and formula of four manometers: simple U-tube, differential, inclined and well-type.
- Calculate Reynolds number (Re = ρvD/μ) and classify a flow as laminar, transitional or turbulent.
- State Bernoulli's theorem, identify its three energy terms, and list its assumptions.
- Apply Bernoulli's principle to the venturi effect, Torricelli's tank-draining law and spray/atomisation.

3.2 Definitions

- **Flow.** The movement of a fluid (a liquid or a gas) from one place to another through a pipe, tube, channel or opening.
- **Fluid.** Any substance that flows. Liquids (water, syrup, oil) and gases (air, steam) are fluids.
- **Flow rate.** How much fluid passes a point in a given time. Two kinds: *volumetric flow rate* (volume per time, e.g. litres per second) and *mass flow rate* (mass per time, e.g. kg per second).
- **Velocity (v).** The speed of the fluid at a point, in metres per second (m/s). In a pipe, fluid near the wall moves slower and fluid in the centre moves faster; v in formulas usually means the *average* velocity.
- **Pressure (P).** The push that the fluid exerts on the walls of the pipe or container, in pascal (Pa) or kilopascal (kPa).
- **Density (ρ, rho).** Mass per unit volume, in kg/m^3. Water ≈ 1000 kg/m^3; mercury ≈ 13 600 kg/m^3.
- **Viscosity (μ, mu).** The "thickness" of a fluid — its resistance to flowing. Water flows easily (low viscosity, μ ≈ 0.001 Pa·s); honey or a thick syrup resists flow (high viscosity).
- **Head.** Pressure expressed as the height of a fluid column, in metres. For example, a pressure head of 2 m of water means the pressure that a 2-metre-tall column of water would exert.

3.3 Terminology

Term	Plain-English meaning
Laminar flow	Smooth, layered flow — fluid moves in neat parallel layers, like cars in separate lanes
Turbulent flow	Chaotic, swirling flow — fluid particles mix randomly, like a crowded market
Transitional flow	In-between state, neither fully smooth nor fully chaotic
Steady flow	Flow properties (velocity, pressure) at a point do not change with time
Unsteady flow	Flow properties at a point change with time
Compressible flow	Fluid density changes noticeably with pressure (typical of gases at high speed)
Incompressible flow	Fluid density stays practically constant (typical of liquids, and gases at low speed)
Viscous flow	Flow where the fluid's internal friction (viscosity) matters
Inviscid flow	An idealisation where viscosity is assumed to be zero (friction ignored)
Volumetric flow rate (Q)	Volume of fluid passing a point per second (m^3/s)
Mass flow rate ($\dot{m}$)	Mass of fluid passing a point per second (kg/s)
Manometer	A device that measures pressure using the height difference of a liquid column
Manometric fluid	The heavy liquid inside a manometer (usually mercury or coloured water/oil)
Reynolds number (Re)	A dimensionless number that predicts whether flow will be laminar or turbulent
Streamline	An imaginary smooth path that a fluid particle follows
Stagnation point	A point where the fluid is brought to a complete stop (velocity = 0)
Head	Pressure expressed as a height of fluid column (metres)
Venturi effect	Pressure falls where a pipe narrows and the fluid speeds up
Atomisation	Breaking a liquid into a fine spray of tiny droplets

3.4 Explanations

3.4.1 Types of flow

Laminar flow. Imagine water flowing very slowly through a thin glass tube. It moves in smooth, parallel layers called *laminae* (Latin for "thin plates"). Each layer slides over its neighbour without mixing. There is no swirling and no crossing of paths.

Turbulent flow. Now turn the tap up fully. The water comes out rough, spluttering and mixing. Particles move in all directions with eddies (small whirlpools). This is turbulent flow — chaotic and well mixed.

Transitional flow. Between laminar and turbulent there is a short unstable region where the flow is neither fully smooth nor fully chaotic. It flickers between the two.

Steady vs unsteady. If you stand at one fixed point in the pipe and the velocity and pressure there stay constant as time passes, the flow is *steady*. If they keep changing with time (for example, when a pump is switched on or off), it is *unsteady*. Note: in steady flow, the velocity can still differ from point to point along the pipe — only the *time* change matters.

Compressible vs incompressible. Liquids are practically *incompressible*: squeezing water barely changes its density, so we treat density as constant. Gases are *compressible*: at high speeds their density changes with pressure. In most pharmacy pipe work (water, syrups, oils), fluids are treated as incompressible.

Viscous vs inviscid. Real fluids have viscosity (internal friction), so their flow is *viscous*. *Inviscid* means "zero viscosity" — a convenient idealisation used in theory (for example, in Bernoulli's theorem) because it simplifies the maths. No real fluid is truly inviscid.

3.4.2 Measurement of flow — volumetric flow rate, mass flow rate, velocity

Volumetric flow rate (Q). Suppose fluid moves with average velocity v (m/s) through a pipe of cross-sectional area A (m^2). In one second, the fluid travels a distance v, so the volume that passes is:

$Q = A \times v$ (units: m^3/s)

For a circular pipe of diameter D: $A = \pi D^2/4$.

> *Worked numerical (a).* A pipe of diameter D = 0.10 m carries water at v = 2.0 m/s. Find Q in m^3/s and L/s.

- Step 1: $A = \pi D^2/4 = 3.1416 \times (0.10)^2 / 4 = 3.1416 \times 0.01 / 4 = 0.007854\ m^2$.
- Step 2: $Q = A \times v = 0.007854 \times 2.0 = 0.015708\ m^3/s$.
- Step 3: Convert to litres per second: 1 m^3 = 1000 L, so $Q = 0.015708 \times 1000 = 15.71$ L/s.

Answer: Q = 0.0157 m^3/s = 15.71 L/s.

Mass flow rate ($\dot{m}$). Mass passing per second. Since mass = density × volume:

$\dot{m} = \rho \times Q = \rho \times A \times v$ (units: kg/s)

Example: for water (ρ = 1000 kg/m^3) with Q = 0.0157 m^3/s above, $\dot{m} = 1000 \times 0.0157 = 15.7$ kg/s. Mass flow rate is used when the exact *weight* of material matters, for example in batch dosing of a syrup.

> **Key takeaway:** Q tells you *how much volume* moves; $\dot{m}$ tells you *how much mass* moves. For incompressible fluids they are linked by density.

3.4.3 Manometers

A manometer measures pressure by balancing it against a column of heavy liquid. The higher the pressure, the bigger the height difference (h) between the two liquid levels.

(i) Simple U-tube manometer — construction. A glass tube bent into a U shape, partly filled with a heavy liquid (usually mercury). One end connects to the point where pressure is measured; the other end is open to the atmosphere (or sealed). A scale is fixed beside the tube to read the height difference h.

Working. Pressure from the pipe pushes the manometric fluid down on one side, so it rises on the other side. At the lowest level of the heavier fluid, pressures balance. The difference in levels, h, is a measure of the pressure difference.

Formula. $\Delta P = (\rho_m - \rho_f) \times g \times h$, where ρ_m = density of manometric fluid, ρ_f = density of the measured fluid, g = 9.81 m/s^2, h = height difference in metres. If the measured fluid is a gas, ρ_f is negligible and $\Delta P \approx \rho_m \times g \times h$.

> *Worked numerical (b).* A U-tube manometer with mercury shows h = 0.25 m. Find ΔP. (ρ_{Hg} = 13 600 kg/m^3, g = 9.81 m/s^2.)

- $\Delta P = \rho \times g \times h = 13\,600 \times 9.81 \times 0.25$.
- $13\,600 \times 9.81 = 133\,416$; $\times 0.25 = 33\,354$ Pa.
- In kPa: 33 354 / 1000 = 33.35 kPa.

Answer: ΔP = 33 354 Pa ≈ 33.35 kPa.

(ii) Differential manometer — construction. A U-tube whose *two* ends connect to *two different points* in the system (for example, the two ends of a venturimeter). Both ends see process pressure; neither is open to the air.

Working. It directly reads the *difference* between two pressures, P1 − P2. The fluid column shifts toward the lower-pressure side.

Formula. ΔP = P1 − P2 = (ρ_m − ρ_f) × g × h.

> *Worked numerical (c).* A differential manometer connects two water pipes. Mercury difference h = 0.12 m. Find ΔP.

- ΔP = h × (ρ_m − ρ_f) × g = 0.12 × (13 600 − 1 000) × 9.81.
- 13 600 − 1 000 = 12 600; 12 600 × 9.81 = 123 606; × 0.12 = 14 832.72 Pa.
- In kPa: 14.83 kPa.

Answer: ΔP ≈ 14 833 Pa ≈ 14.83 kPa.

(iii) Inclined manometer — construction. Like a U-tube, but one leg is tilted at a small angle θ to the horizontal, with a scale along the tilted tube. It contains a light manometric fluid (coloured oil or water).

Working. For the same vertical height h, the liquid travels a much longer distance L along the slope (L = h / sin θ). A small pressure change therefore produces a long, easily readable movement — it magnifies the reading.

Formula. True vertical height h = L × sin θ, so ΔP = ρ × g × L × sin θ.

Example: if L = 0.20 m at θ = 15°, h = 0.20 × sin 15° = 0.20 × 0.2588 = 0.0518 m. With oil of ρ = 900 kg/m^3: ΔP = 900 × 9.81 × 0.0518 ≈ 457 Pa — a small pressure read comfortably on a 20 cm scale.

(iv) Well-type (cistern) manometer — construction. One side is a wide reservoir ("well") and the other side is a single narrow vertical tube open to the atmosphere. Only one liquid level needs to be read.

Working. Pressure pushes liquid from the well up the narrow tube. Because the well is very wide, its level barely falls, so nearly the full height change appears in the narrow tube — one reading gives h directly.

Formula. ΔP ≈ ρ × g × h (with a small correction for the well's area if high accuracy is needed: true h = h_read × (1 + a/A), where a = tube area, A = well area; usually neglected).

> **Key takeaway:** U-tube is the basic design; differential reads the *difference between two points*; inclined magnifies small pressures; well-type needs only one reading.

3.4.4 Reynolds number and its significance

The idea. Osborne Reynolds showed that whether flow is smooth or chaotic depends on a competition: *inertia* (the fluid's tendency to keep moving, ρvD) versus *viscosity* (the fluid's internal friction, μ). Their ratio is dimensionless — the units cancel out:

Re = (ρ × v × D) / μ

- ρ = density (kg/m^3), v = average velocity (m/s), D = pipe diameter (m), μ = dynamic viscosity (Pa·s).
- "Dimensionless" means Re is a pure number with no units — check: (kg/m^3)(m/s)(m) / (kg/m·s) = 1. Because it has no units, it works in any unit system.

Derivation idea (not a full proof). Inertial force on the fluid ~ mass × acceleration ~ ρ × v^2 × D^2. Viscous (friction) force ~ μ × v × D. Divide inertial by viscous: (ρv^{2}D^2)/(μvD) = ρvD/μ = Re. So Re literally measures "how strongly inertia beats friction".

Critical values (for flow inside a pipe):

- Re < 2100 → **laminar** flow (smooth layers).
- 2100 < Re < 4000 → **transitional** flow (unstable, flickering).
- Re > 4000 → **turbulent** flow (chaotic mixing).

> *Worked numerical (d).* Water: ρ = 1000 kg/m^3, μ = 0.001 Pa·s, v = 1.5 m/s, D = 0.05 m. Find Re and classify.

- Re = ρvD/μ = 1000 × 1.5 × 0.05 / 0.001.
- Numerator: 1000 × 1.5 × 0.05 = 75. Divide by 0.001 → 75 000.
- 75 000 > 4000 → **turbulent flow**.

> *Worked numerical (e).* Oil: ρ = 880 kg/m^3, μ = 0.12 Pa·s, v = 0.20 m/s, D = 0.03 m. Find Re and classify.

- Re = 880 × 0.20 × 0.03 / 0.12.
- Numerator: 880 × 0.20 × 0.03 = 5.28. Divide by 0.12 → 44.
- 44 < 2100 → **laminar flow**.

Significance in pharma piping. Re predicts mixing and behaviour without expensive trials:

- Laminar flow (low Re) is gentle — good for shear-sensitive products like emulsions and suspensions that would break or separate if churned.
- Turbulent flow (high Re) mixes well — good for cleaning pipes (CIP), blending, and heat transfer.
- Re guides pump selection and pipe sizing: a thick syrup needs a wider pipe or slower speed to keep Re (and pressure drop) manageable.
- Many equipment correlations (pressure drop, heat transfer, mass transfer) use Re as their key input.

> **Key takeaway:** Re = ρvD/μ. Below 2100 smooth, above 4000 chaotic, between 2100–4000 unstable. It tells the engineer what the fluid will do before a single drop flows.

3.4.5 Bernoulli's theorem and its applications

Statement. For the steady flow of an incompressible, inviscid (frictionless) fluid along a streamline, the total mechanical energy per unit weight stays constant:

$P/(\rho g) + v^2/(2g) + z = \text{constant}$

The three energy terms (all measured in metres of head):

1. **Pressure head** = P/(ρg). Energy due to pressure — like the push stored in a squeezed pipe.
2. **Velocity head** = $v^2/(2g)$. Energy due to motion — kinetic energy per unit weight.
3. **Elevation head** = z. Energy due to height above a reference level — potential energy per unit weight.

Meaning in plain words. Energy is conserved: if the fluid speeds up (velocity head rises), the pressure must fall (pressure head drops) to keep the total constant — provided height does not change. Faster flow = lower pressure. This single idea drives venturimeters, spray guns and many pharma devices.

Assumptions (conditions under which the theorem holds):

1. Flow is steady (no change with time at a point).
2. Fluid is incompressible (density constant — true for liquids).
3. Fluid is inviscid (no friction/viscosity losses — an idealisation; real calculations add a loss term).
4. Flow is along a single streamline (no mixing across layers).
5. No pump adds energy and no turbine removes energy between the two points.

> *Worked numerical (f).* Horizontal pipe (z1 = z2), water: P1 = 200 kPa, v1 = 1.0 m/s, v2 = 3.0 m/s. Find P2.

- Bernoulli with z1 = z2: $P1/(\rho g) + v1^2/(2g) = P2/(\rho g) + v2^2/(2g)$.
- Multiply through by ρg: $P1 + \frac{1}{2}\rho v1^2 = P2 + \frac{1}{2}\rho v2^2$.
- $P2 = P1 + \frac{1}{2}\rho(v1^2 - v2^2) = 200\,000 + 0.5 \times 1000 \times (1^2 - 3^2)$.
- $= 200\,000 + 500 \times (1 - 9) = 200\,000 - 4\,000 = 196\,000$ Pa.

Answer: P2 = 196 kPa. Notice: velocity rose from 1 to 3 m/s, so pressure fell from 200 to 196 kPa — exactly as Bernoulli predicts.

Applications.

1. **Venturi principle.** Where a pipe narrows (throat), velocity rises and pressure falls. This pressure drop is measured (by a manometer) to calculate flow rate — the working basis of the venturimeter and orifice meter (Chapter 4).
2. **Torricelli's law (tank draining).** An open tank with liquid depth h above a small outlet: apply Bernoulli between the top surface (P = atmospheric, v ≈ 0, z = h) and the outlet (P = atmospheric, z = 0). The result: $v = \sqrt{(2gh)}$. The exit speed depends only on the depth, not on the liquid's density — the same as a freely falling body.

> *Worked numerical (g).* Open water tank, depth h = 4.0 m above the outlet. Exit velocity?

- $v = \sqrt{(2gh)} = \sqrt{(2 \times 9.81 \times 4.0)} = \sqrt{78.48} = 8.86$ m/s.

Answer: v ≈ 8.86 m/s.

3. **Spray/atomisation.** A fast air jet blown across a liquid tube lowers the pressure at the tube tip (Bernoulli). Outside atmospheric pressure then pushes the liquid up the tube, and the air jet shatters it into fine droplets — the principle of perfume atomisers, nebulisers and spray dryers used in pharma.

> **Key takeaway:** Bernoulli says pressure head + velocity head + elevation head = constant. Speed up → pressure drops. This trade-off is the engine behind flow meters and sprays.

3.5 Classifications

Classification of flow (summary tree):

- By pattern of particle motion:
- Laminar (Re < 2100) — orderly layers
- Transitional (2100–4000) — unstable
- Turbulent (Re > 4000) — chaotic eddies
- By time dependence:
- Steady — properties at a point constant in time
- Unsteady — properties at a point change with time
- By density behaviour:
- Incompressible — density constant (liquids; gases at low speed)
- Compressible — density changes (gases at high speed)
- By friction treatment:
- Viscous — viscosity included (real fluids)
- Inviscid — viscosity assumed zero (theory/idealisation)

Classification of manometers (by use):

- Simple U-tube — one point vs atmosphere
- Differential — difference between two points
- Inclined — magnified small pressures
- Well-type — single-reading gauge pressures

3.6 Mechanisms (step-by-step)

Mechanism 1: How a U-tube manometer balances pressure.

1. Connect one limb to the pressure point; leave the other open to air.
2. The higher pipe pressure pushes the mercury down in the connected limb.
3. Mercury rises in the open limb until the weight of the extra column exactly balances the extra pressure.
4. Read the height difference h between the two levels.
5. Compute ΔP = (ρ_m − ρ_f)·g·h. The heavy liquid's weight *is* the pressure scale.

Mechanism 2: How Bernoulli's energy trade works in a narrowing pipe.

1. Pipe narrows → same volume must pass a smaller area per second (continuity: A1·v1 = A2·v2).
2. Velocity rises in the narrow section (v2 > v1).
3. Velocity head $v^2/2g$ rises.
4. Total head is constant (Bernoulli) and elevation is unchanged, so pressure head P/ρg must fall.
5. Result: the narrow throat shows lower pressure — measured by a manometer, this pressure drop reveals the flow rate.

Mechanism 3: Torricelli's tank draining, step-by-step.

1. Pick point 1 at the liquid surface (large tank, so v1 ≈ 0; open, so P1 = atmospheric).
2. Pick point 2 at the outlet (P2 = atmospheric; elevation z2 = 0; surface at z1 = h).
3. Bernoulli: Patm/ρg + 0 + h = Patm/ρg + $v2^2/2g$ + 0.
4. Atmospheric terms cancel: h = $v2^2/2g$ → v2 = √(2gh).
5. The outlet behaves like a body dropped from height h — depth alone fixes the speed.

3.7 Diagrams (textual, labelled — draw from these descriptions)

Diagram 3.1 — Simple U-tube manometer. Draw a "U" of glass tubing. Label: (1) left limb connected to pressure source (pipe tap); (2) right limb open to atmosphere; (3) mercury (shaded) filling the bottom bend; (4) lower mercury level on the high-pressure side; (5) higher mercury level on the atmosphere side; (6) vertical double-arrow between the two levels marked "h = height difference"; (7) scale in cm beside the right limb. Title: "Simple U-tube manometer — pressure pushes fluid down one side, up the other".

Diagram 3.2 — Differential manometer. Draw a "U" with both limbs rising to pipe connections. Label: (1) left limb to pressure point P1 (e.g., before a constriction); (2) right limb to pressure point P2 (e.g., after a constriction); (3) mercury in the bend; (4) level difference h between limbs (higher on the P2/low-pressure side); (5) note "neither limb open to air". Title: "Differential manometer — reads P1 − P2 directly".

Diagram 3.3 — Inclined manometer. Draw a small reservoir on the left joined to a long tube tilted at angle θ above horizontal. Label: (1) reservoir (well) connected to pressure source; (2) inclined glass tube at angle θ; (3) coloured oil inside; (4) meniscus position marked L along the slope; (5) angle θ marked at the base; (6) vertical dashed line showing true height h = L·sinθ; (7) note "small pressure → long readable length L". Title: "Inclined manometer — slope magnifies small pressures".

Diagram 3.4 — Bernoulli energy heads along a pipe. Draw a horizontal pipe that narrows in the middle (throat). Above it draw three stacked bars at section 1 (wide) and section 2 (throat). Label bars: (1) pressure head P/ρg (tall at section 1, short at section 2); (2) velocity head $v^2/2g$ (short at section 1, tall at section 2); (3) elevation head z (same at both, pipe horizontal); (4) total head line — a flat dashed line across both sections marked "constant". Title: "Bernoulli — as velocity head rises, pressure head falls; total stays constant".

3.8 Flowcharts

Flowchart 3.1 — Is the flow laminar or turbulent?

```
START
  |
  v
Collect: rho (density), v (velocity), D (pipe diameter), mu (viscosity)
  |
  v
Compute Re = rho * v * D / mu
  |
  v
Is Re < 2100? ----YES----> LAMINAR (smooth layers)
  |
  NO
  |
  v
Is Re < 4000? ----YES----> TRANSITIONAL (unstable)
  |
  NO
  |
  v
TURBULENT (chaotic mixing)
```

Flowchart 3.2 — Which manometer to use?

```
START: what do you need to measure?
  |
  +-- One pressure vs atmosphere? --> gauge range moderate?
  |       YES --> Simple U-tube manometer
  |       NO (very small) --> Inclined or well-type manometer
  |
  +-- Difference between TWO points? --> Differential manometer
  |
  +-- Tiny pressure (draught, filter)? --> Inclined manometer
```

3.9 Infographic concept box

```
┌─ INFOGRAPHIC: THE FLOW TRIANGLE ──────────────────────────┐
│ Draw a triangle with three corners: │
│ │
│ SPEED (v) ◄──────────► PRESSURE (P) │
│ \ / │
│ \ ENERGY IS / │
│ \ CONSERVED / │
│ \ / │
│ ▼ ▼ │
│ HEIGHT (z) │
│ │
│ Centre label: "Bernoulli: P/ρg + v²/2g + z = constant" │
│ Side notes: "Narrow pipe → v up → P down (venturi)" · │
│ "Re = ρvD/μ decides smooth (laminar) vs chaotic (turbulent)" · │
│ "Manometer: heavy liquid column weighs the pressure" │
└──────────────────────────────────────────────────────────┘
```

3.10 Tables

Table 3.1 — Flow types compared.

Feature	Laminar flow	Transitional flow	Turbulent flow
Reynolds number	< 2100	2100–4000	> 4000
Particle paths	Smooth parallel layers	Unstable, flickering	Chaotic, swirling eddies
Mixing	Very little	Partial	Excellent
Pressure drop	Low	Moderate	High
Example	Thick syrup in a pipe	Flow just after a valve opens	Water from a fully open tap
Pharma use	Gentle handling of emulsions	Avoid — unpredictable	Blending, CIP cleaning

Table 3.2 — Manometer types compared.

Manometer	Measures	Construction	Formula	Best for
Simple U-tube	One pressure vs atmosphere	U-tube, mercury, scale	$\Delta P = (\rho_m - \rho_f)gh$	General gauge pressure
Differential	Difference P1 − P2	U-tube, both ends to process	$\Delta P = (\rho_m - \rho_f)gh$	Across filters, meters
Inclined	Small pressures	Tilted tube at angle θ	$\Delta P = \rho g L \cdot \sin\theta$	Draughts, tiny ΔP
Well-type	Gauge pressure	Wide well + one narrow tube	$\Delta P \approx \rho gh$	Single quick reading

Table 3.3 — The three Bernoulli heads.

Head	Formula	Meaning	Units
Pressure head	$P/(\rho g)$	Energy stored as pressure	m
Velocity head	$v^2/(2g)$	Energy of motion	m
Elevation head	z	Energy of height	m
Total head	Sum of the three	Constant along a streamline	m

3.11 Examples

- **Syrup transfer:** A cough-syrup line (thick, viscous) runs at low velocity in a narrow pipe. Re works out below 2100 → laminar. The syrup glides in layers without frothing — ideal, because turbulence would trap air bubbles in the bottle.
- **CIP cleaning:** During clean-in-place, water is pumped fast through the same pipes. Re shoots above 4000 → turbulent. The chaotic scrubbing action strips residues off the pipe walls.
- **Tank draining:** A 4 m deep purified-water storage tank drains through a bottom outlet. Torricelli gives $v = \sqrt{(2 \times 9.81 \times 4)} \approx 8.86$ m/s at the outlet — the operator uses this to size the drain valve.
- **Filter monitoring:** A differential manometer across a sterile filter shows a slowly rising ΔP. Rising ΔP means the filter is clogging — maintenance is scheduled before the batch is affected.
- **Venturi suction in a spray dryer:** High-speed air through a narrow throat creates low pressure that sucks feed liquid in and atomises it into droplets for drying.

3.12 Analogies

Laminar vs turbulent = traffic. Laminar flow is cars moving smoothly in marked lanes on an empty highway. Turbulent flow is a crowded bazaar road at rush hour — autos, bikes and pedestrians weaving and swirling in every direction.

Bernoulli = crowd through a gate. A crowd (fluid) walking through a wide corridor moves slowly and pushes hard on the walls (high pressure). Force the same crowd through a narrow gate and they speed up — but they push sideways less (low pressure). Faster through the narrow part, less sideways push.

Manometer = weighing scale for pressure. Just as a kitchen scale weighs vegetables by balancing them against a spring, a manometer "weighs" pressure by balancing it against a column of heavy liquid.

Reynolds number = referee. Before a match, a referee decides the rules. Re is the referee that decides whether the fluid will play the "smooth game" (laminar) or the "rough game" (turbulent).

Torricelli = falling stone. Water shooting from a tank outlet at depth h leaves with the same speed as a stone dropped from height h — $v = \sqrt{(2gh)}$ in both cases.

3.13 Pharmacy relevance

- **Piping design:** Every liquid line in a pharma plant — purified water loops, syrup transfer, solvent lines — is sized using Re and Bernoulli so that pressure drops stay within pump limits.
- **Sterile filtration:** Differential manometers/gauges across membrane filters track clogging (rising ΔP) without opening the sterile line.
- **Spray drying and coating:** Bernoulli's venturi suction atomises coating solutions onto tablets; droplet size decides coating quality.
- **Nebulisers and inhalers:** The same venturi principle turns liquid drug into a breathable mist.
- **Gentle product handling:** Emulsions, suspensions and protein solutions are kept in laminar flow to avoid shear damage and foaming.
- **Tank and vessel draining:** Torricelli's law estimates drain times for formulation vessels and storage tanks.

3.14 India context

- Indian pharmaceutical plants — from large formulation units in Hyderabad, Ahmedabad and Baddi to smaller units across the country — run extensive purified-water (PW) distribution loops. These loops are typically kept in continuous turbulent circulation, a design choice guided by Reynolds-number thinking, because stagnant or slow-moving water risks microbial growth.
- Differential pressure gauges (the industrial cousins of differential manometers) are a familiar sight across cleanroom doors in Indian sterile plants; they confirm that higher-grade rooms stay at positive pressure relative to corridors.
- Inclined manometers are still found in older Indian plant utility areas for reading the very small draught pressures of ventilation ducts and boiler flues.
- Spray dryers used for drying pharmaceutical granules and excipients in Indian bulk-drug units rely on atomisation — the Bernoulli-based breakup of liquid feed into droplets.
- Mercury manometers are being phased out in favour of digital gauges in many facilities because of mercury's toxicity; the principles in this chapter apply unchanged to the digital instruments.

3.15 Key points

- $Q = A \times v$ (m^3/s); $\dot{m} = \rho \times Q$ (kg/s); $A = \pi D^2/4$ for a circular pipe.
- Laminar < 2100 < transitional < 4000 < turbulent (Reynolds number in pipes).
- $Re = \rho v D/\mu$ is dimensionless: inertia ÷ viscosity.
- Manometers measure pressure as a liquid-column height: $\Delta P = (\rho_m - \rho_f) \cdot g \cdot h$.
- Differential manometer reads P1 − P2 directly; inclined magnifies small ΔP via $L \cdot \sin\theta$.
- Bernoulli: $P/\rho g + v^2/2g + z$ = constant (steady, incompressible, inviscid, along a streamline).
- Faster flow → lower pressure (venturi effect).
- Torricelli: $v = \sqrt{(2gh)}$ for a tank outlet.
- Bernoulli applications: venturi suction, tank draining, spray/atomisation (nebulisers, spray dryers).

3.16 Common confusions

- **Q vs v:** Q (m^3/s) is the *volume per second*; v (m/s) is the *speed*. They are linked by Q = A·v, not equal. A narrow pipe can have high v but small Q.
- **Laminar vs steady:** Laminar describes the *pattern* (smooth layers); steady describes *time behaviour* (unchanging). A flow can be turbulent yet steady (a fully open tap), or laminar yet unsteady.
- **Gauge vs differential:** A simple U-tube compares one pressure with the *atmosphere*; a differential manometer compares *two process pressures* with each other.
- **Head vs pressure:** Head is in *metres* (a height); pressure is in *pascal*. They are related by P = ρgh but are not the same quantity.
- **Inviscid vs viscous:** Inviscid (zero viscosity) is a *theoretical assumption* for Bernoulli, not a property of real fluids. All real fluids are viscous.
- **2100 vs 4000:** Below 2100 is laminar; above 4000 is turbulent; 2100–4000 is transitional. Do not call 3000 "laminar".

3.17 Common mistakes

- Using diameter instead of radius (or vice versa) in $A = \pi D^2/4$ — with D it is $D^2/4$, with radius r it is πr^2. Mixing them gives a 4× error.
- Forgetting unit conversion: h in cm must become metres before using ΔP = ρgh; answers in Pa, not kPa, unless divided by 1000.
- Dropping the (ρ_m − ρ_f) term in differential manometers — when both sides contain the process fluid, subtract its density.
- Applying Bernoulli across a pump or a valve — the theorem assumes no added/removed energy and no friction losses between the two points.
- Using gauge pressure at one point and absolute at another in the same equation — be consistent.
- Reading an inclined manometer's L as h — the true vertical height is L·sinθ, always smaller than L.
- Assuming turbulent flow is "bad" — in cleaning and mixing, turbulence is exactly what you want.

3.18 Mnemonics

- **Re thresholds — "21-40 lane rule":** *"Below 21 (hundred) stay in your lane"* (laminar, < 2100); *"above 40 (hundred) it's a free-for-all"* (turbulent, > 4000); between is the *transition jam*.
- **Bernoulli heads — "PVZ" / "Pressure, Velocity, Zee(height)":** *"PVZ — Push, Velocity, Zee"* — the three heads that sum to a constant.
- **Manometer types — "S-D-I-W":** *"Single, Double, Inclined, Well"* — Single point (U-tube), Double point (differential), Inclined (tiny pressures), Well (one reading).
- **Torricelli — "Two-g-h root":** *"The outlet speed is the root of two-g-h"* — $v = \sqrt{2gh}$.

3.19 Exam summary

Flow is classified by pattern (laminar < 2100, transitional 2100–4000, turbulent > 4000 via Re = ρvD/μ), by time (steady/unsteady), by density (incompressible/compressible) and by friction treatment (viscous/inviscid). Volumetric flow rate Q = A·v ($A = \pi D^2/4$); mass flow rate $\dot{m} = \rho Q$. Manometers balance pressure against a liquid column: simple U-tube (one point vs air), differential (P1 − P2), inclined ($\Delta P = \rho g L \cdot \sin\theta$ for tiny pressures), well-type (single reading). Bernoulli's theorem — $P/\rho g + v^2/2g + z$ = constant for steady, incompressible, inviscid flow along a streamline — means faster flow gives lower pressure; applications include the venturi effect, Torricelli's tank-draining law $v = \sqrt{2gh}$, and spray/atomisation in nebulisers and spray dryers.

3.20 One-minute revision

- Q = Av; $\dot{m}$ = ρQ. Units: m^3/s and kg/s.
- Re = ρvD/μ — < 2100 laminar, 2100–4000 transitional, > 4000 turbulent.
- Manometer formula: ΔP = (ρ_m − ρ_f)gh. Inclined: use L·sinθ.
- Bernoulli: pressure head + velocity head + elevation head = constant.
- Narrow pipe → fast flow → low pressure.
- Torricelli: v = √(2gh).
- Laminar = gentle (emulsions); turbulent = mixing/cleaning.

3.21 Exam questions

Long/essay questions (5):

1. Derive the expression for Reynolds number and explain its significance in pharmaceutical piping. Illustrate with the critical values 2100 and 4000.
2. State Bernoulli's theorem. Explain the three energy terms with units, list all assumptions, and derive Torricelli's law for tank draining.
3. Describe the construction and working of a differential manometer. How does it differ from a simple U-tube manometer? Derive the pressure-difference formula.
4. Classify the different types of flow with neat sketches. Explain steady vs unsteady and compressible vs incompressible flow with one pharma example each.
5. Explain the venturi effect using Bernoulli's theorem. Describe any two applications — spray/atomisation and one flow-measuring device.

Short questions (8):

1. Define volumetric flow rate and mass flow rate. Give units.
2. What is the difference between laminar and turbulent flow?
3. Why is Reynolds number called dimensionless? Show the unit cancellation.
4. Write the formula of a simple U-tube manometer and name each symbol.
5. Why is an inclined manometer preferred for very small pressures?
6. State the five assumptions of Bernoulli's theorem.
7. What is Torricelli's law? Write its formula.
8. Give two pharmacy applications of the venturi principle.

MCQs (10) — answer key at the end:

1. The SI unit of volumetric flow rate is —
   (a) m/s (b) m^3/s (c) kg/s (d) Pa
2. Reynolds number is the ratio of —
   (a) viscous force to inertial force (b) inertial force to viscous force (c) pressure force to gravity force (d) gravity force to pressure force
3. Flow with Re = 1500 in a pipe is —
   (a) turbulent (b) transitional (c) laminar (d) unsteady
4. Flow with Re = 6000 in a pipe is —
   (a) laminar (b) transitional (c) turbulent (d) inviscid
5. In a differential manometer, the height difference h directly measures —
   (a) absolute pressure (b) atmospheric pressure (c) pressure difference P1 − P2 (d) velocity
6. An inclined manometer is used mainly for —
   (a) very high pressures (b) very small pressures (c) corrosive fluids (d) high temperatures
7. In Bernoulli's equation, $v^2/2g$ represents —
   (a) pressure head (b) velocity head (c) elevation head (d) friction head
8. According to Bernoulli, when fluid speeds up in a horizontal pipe, the pressure —
   (a) increases (b) decreases (c) stays same (d) becomes zero
9. Torricelli's law gives the outlet velocity as —
   (a) √(gh) (b) √(2gh) (c) 2gh (d) gh/2
10. Atomisation in a spray dryer works on the principle of —
    (a) Reynolds number (b) Bernoulli's theorem (c) Torricelli's law (d) Pascal's law

Answer key: 1-(b), 2-(b), 3-(c), 4-(c), 5-(c), 6-(b), 7-(b), 8-(b), 9-(b), 10-(b).

Viva questions (5) with brief answers:

1. *What does a Reynolds number below 2100 tell you?* — The flow is laminar: smooth, layered, with little mixing.
2. *Why is mercury commonly used in manometers?* — It is very dense (13 600 kg/m^3), so even large pressures give short, readable columns; it does not wet glass.
3. *What is the physical meaning of "head" in metres?* — Pressure or energy expressed as the height of a fluid column that would produce it; P = ρgh.
4. *Can Bernoulli's theorem be applied across a pump?* — No. The theorem assumes no energy is added or removed between the two points; a pump adds energy.
5. *Why does a perfume atomiser spray when you squeeze the bulb?* — Fast air past the tube tip lowers pressure (Bernoulli); outside air pressure pushes liquid up, and the jet breaks it into droplets.

Application-based questions (3):

1. A purified-water loop in a formulation plant shows Re = 85 000. The QA engineer worries about microbial growth in slow zones. Is the flow regime helping or hurting, and why?
2. A differential gauge across a sterile filter reads a steadily increasing ΔP over three batches. Using manometer principles, explain what is happening and what action you would recommend.
3. A viscous ointment base must be transferred without trapping air. Would you design for laminar or turbulent flow? Justify using Reynolds-number reasoning.

3.22 References

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CHAPTER 4 — FLOW MEASUREMENT DEVICES: Orifice Meter, Venturimeter, Pitot Tube, Rotameter

4.1 Learning objectives

After studying this chapter, the student will be able to:

- Explain the common working principle (Bernoulli + continuity) behind the orifice meter, venturimeter, pitot tube and rotameter.
- Describe the construction and working of each of the four devices with labelled diagrams.
- Write and apply the flow-rate formula of each device, including the discharge coefficients Co and Cd.
- Calculate flow rate from orifice-meter and venturimeter readings, velocity from a pitot tube, and flow from a rotameter scale.
- State the advantages of the venturimeter over the orifice meter.
- Explain how a rotameter is read and what calibration means.
- Compare the four devices in a table and select a suitable meter for a given fluid and duty.

4.2 Definitions

- **Orifice meter.** A flow meter that measures flow rate from the pressure drop across a thin plate with a hole (orifice) fixed inside the pipe.
- **Venturimeter.** A flow meter that measures flow rate from the pressure drop across a smoothly converging–diverging pipe section.
- **Pitot tube.** A device that measures fluid *velocity* at a point by sensing stagnation pressure (the pressure when moving fluid is brought to rest).
- **Rotameter.** A variable-area flow meter: a float inside a tapered vertical tube rises to a height that indicates flow rate on a scale.
- **Coefficient of discharge (Co, Cd).** A correction factor (< 1) that accounts for friction and real-flow effects; actual flow = coefficient × theoretical flow.
- **Beta ratio (β).** In an orifice meter, β = orifice diameter / pipe diameter (d/D).
- **Vena contracta.** The narrowest point of the fluid jet just downstream of an orifice, where velocity is maximum.
- **Stagnation pressure.** The pressure at a point where the fluid is brought completely to rest; equals static pressure + dynamic pressure.
- **Calibration.** Checking and marking a measuring instrument against a known standard so its readings are trustworthy.

4.3 Terminology

Term	Plain-English meaning
Orifice	A precisely cut hole in a plate placed across the pipe
Throat	The narrowest section of a venturimeter
Convergent cone	The gradually narrowing entry part of a venturimeter
Divergent cone	The gradually widening exit part of a venturimeter
Tapping	A small hole/port where a pressure line or manometer connects
Coefficient of discharge	Real flow ÷ ideal (frictionless) flow; always less than 1
Vena contracta	The pinched-in narrowest part of the jet after an orifice
Stagnation point/pressure	Where moving fluid stops dead; its pressure is maximum
Static pressure	Pressure of the fluid as it flows past (sideways push)
Dynamic pressure	Extra pressure due to the fluid's motion ($\frac{1}{2}\rho v^2$)
Float (rotameter)	The shaped bob that rides up and down in the tapered tube
Variable area	Meter type where the flow passage area changes with flow rate
Range/turndown	The span of flow rates a meter can measure accurately

4.4 Explanations

4.4.1 The common principle — Bernoulli + continuity

Three of the four devices (orifice meter, venturimeter, pitot tube) work on one shared idea:

1. **Continuity:** what flows in must flow out — A1·v1 = A2·v2. Force fluid through a smaller area and it must speed up.
2. **Bernoulli:** faster flow means lower pressure (in a horizontal pipe).
3. So a constriction creates a **pressure drop (ΔP)** that grows with flow rate. Measure ΔP with a manometer, and you can calculate Q.

The rotameter is different: instead of a fixed constriction with changing ΔP, it keeps ΔP nearly constant and lets the *area* change — a float rises until the opening around it is just big enough for the flow.

> **Key takeaway:** Orifice, venturi and pitot translate "how much the pressure changed" into flow rate. The rotameter translates "how high the float sits" into flow rate.

4.4.2 Orifice meter — construction, working, formula

Construction. A thin flat metal plate (the *orifice plate*) with a sharp-edged circular hole (the *orifice*, diameter d) is clamped between two pipe flanges, square across the pipe of diameter D. Two pressure tappings connect to a differential manometer: one just upstream of the plate, one just downstream (at or near the vena contracta).

Working.

1. Fluid approaches the plate at velocity v1 and pressure P1.
2. It squeezes through the small hole; the jet contracts further to the *vena contracta* downstream, where velocity is maximum and pressure minimum.
3. The differential manometer reads the pressure drop as a head difference Δh.
4. Because the hole is sharp-edged, extra turbulence and friction occur — so the real flow is less than the ideal Bernoulli value. The *coefficient of discharge* Co (typically ~0.6–0.65) corrects for this.

Formula.

$Q = Co \cdot Ao \cdot \sqrt{(2g \cdot \Delta h)} / \sqrt{(1 - \beta^4)}$

- $Ao = \pi d^2/4$ (area of the orifice), $\beta = d/D$, Δh = manometer reading converted to head of the *flowing* fluid: $\Delta h = h \cdot (\rho_m/\rho_f - 1)$, where h is the manometric fluid column height.
- $Co \approx 0.62$ (given in problems or found by calibration).

> *Worked numerical (i).* Pipe D = 0.10 m, orifice d = 0.05 m, mercury h = 0.15 m, Co = 0.62, water flowing. Find Q.

- Step 1: $\beta = d/D = 0.05/0.10 = 0.5$; $\beta^4 = 0.0625$.
- Step 2: $Ao = \pi d^2/4 = 3.1416 \times 0.0025 / 4 = 0.0019635\ m^2$.
- Step 3: $\Delta h = h(\rho_m/\rho_f - 1) = 0.15 \times (13\,600/1\,000 - 1) = 0.15 \times 12.6 = 1.89$ m of water.
- Step 4: $\sqrt{(2g\Delta h)} = \sqrt{(2 \times 9.81 \times 1.89)} = \sqrt{37.08} = 6.0896$ m/s.
- Step 5: $\sqrt{(1 - \beta^4)} = \sqrt{(1 - 0.0625)} = \sqrt{0.9375} = 0.96825$.
- Step 6: $Q = 0.62 \times 0.0019635 \times 6.0896 / 0.96825 = 0.00766\ m^3/s$.
- In L/s: $0.00766 \times 1000 = 7.66$ L/s.

Answer: Q ≈ 0.00766 m^3/s = 7.66 L/s.

4.4.3 Venturimeter — construction, working, formula, advantages over orifice

Construction. A short pipe section with three parts in series: (1) a *convergent cone* that narrows smoothly from pipe diameter D1 to throat diameter D2 (typical angle ~15–20°); (2) a short cylindrical *throat*; (3) a *divergent cone* that widens gradually back to D1 (shallow angle ~5–7° to avoid flow separation). Pressure tappings at the inlet (section 1) and the throat (section 2) connect to a differential manometer.

Working.

1. Fluid enters the convergent cone and accelerates smoothly toward the throat (continuity).
2. At the throat, velocity is maximum and pressure minimum (Bernoulli).
3. The divergent cone slows the fluid gently, recovering most of the pressure.
4. The manometer reads Δh between inlet and throat; Q follows from the formula with coefficient of discharge Cd (typically ~0.95–0.98 — much closer to 1 than an orifice plate, because the smooth shape wastes little energy).

Formula.

$Q = Cd \cdot A1 \cdot A2 \cdot \sqrt{(2g \cdot \Delta h)} / \sqrt{(A1^2 - A2^2)}$

- $A1 = \pi D1^2/4$ (pipe), $A2 = \pi D2^2/4$ (throat), $\Delta h = h(\rho_m/\rho_f - 1)$ as before.

> *Worked numerical (h).* D1 = 0.15 m, D2 = 0.075 m, mercury h = 0.20 m, Cd = 0.97, water flowing. Find Q.

- Step 1: $A1 = \pi(0.15)^2/4 = 0.017671\ m^2$; $A2 = \pi(0.075)^2/4 = 0.0044179\ m^2$.
- Step 2: $\Delta h = 0.20 \times (13\,600/1\,000 - 1) = 0.20 \times 12.6 = 2.52$ m of water.
- Step 3: $\sqrt{(2g\Delta h)} = \sqrt{(2 \times 9.81 \times 2.52)} = \sqrt{49.44} = 7.0316$ m/s.
- Step 4: $\sqrt{(A1^2 - A2^2)} = \sqrt{(0.00031228 - 0.000019518)} = \sqrt{0.00029276} = 0.017111\ m^2$.
- Step 5: $Q = 0.97 \times 0.017671 \times 0.0044179 \times 7.0316 / 0.017111 = 0.03112\ m^3/s$.
- In L/s: 31.12 L/s.

Answer: Q ≈ 0.0311 m^3/s = 31.1 L/s.

Advantages of the venturimeter over the orifice meter:

1. **Lower permanent pressure loss** — the gradual divergent cone recovers pressure; the orifice plate wastes energy in turbulence.
2. **Higher discharge coefficient** (Cd ≈ 0.97 vs Co ≈ 0.62) — readings are closer to theory and more accurate.
3. **Less wear and clogging** — no sharp edge to erode; suspended solids pass through more easily, suiting slurries.
4. **Works better with viscous fluids** — smooth contours keep the flow orderly.
5. Main drawback: it is longer, bulkier and more expensive than a simple orifice plate.

4.4.4 Pitot tube — construction, working, formula

Construction. A slim L-shaped tube: one opening faces *directly into* the flowing fluid (facing upstream), and a second opening (or a concentric outer tube with side holes) senses the *static* pressure of the stream. The two legs connect to a differential manometer.

Working.

1. Fluid rams into the forward-facing opening and is brought to rest — this is the *stagnation point*. Its pressure (stagnation pressure) = static pressure + dynamic pressure ($\frac{1}{2}\rho v^2$).
2. The side opening reads static pressure alone.
3. The manometer shows the difference = dynamic pressure head Δh.
4. From Bernoulli: $\frac{1}{2}\rho v^2 = \rho g \Delta h \rightarrow v = \sqrt{(2g \cdot \Delta h)}$.

Formula (velocity at the tube tip).

$v = \sqrt{(2g \cdot \Delta h)}$

- Δh = manometer reading expressed as head of the flowing fluid. (Multiply by a probe coefficient ≈ 1 if given.)

> *Worked numerical (j).* Water flow; manometer Δh = 0.05 m of water. Find v.

- $v = \sqrt{(2 \times 9.81 \times 0.05)} = \sqrt{0.981} = 0.9905$ m/s.

Answer: v ≈ 0.99 m/s.

Note: the pitot tube measures *local velocity at one point* (usually the pipe centre, where it is maximum), not the average flow rate directly. To get Q, measure velocity at several points across the pipe and average, then Q = A × v_avg.

4.4.5 Rotameter — variable-area meter; construction, working, reading, calibration

Construction. A vertical glass (or metal) tube that is *tapered* — narrow at the bottom, wider at the top — with a flow scale marked on it. Inside sits a specially shaped *float* (often with grooves that make it spin steadily). Fluid enters at the bottom and flows upward around the float, leaving at the top.

Working.

1. At zero flow the float rests at the bottom on a stop.
2. As flow increases, the upward drag of the fluid lifts the float.
3. The float rises into the wider part of the tube, so the annular (ring-shaped) gap around it grows.
4. It settles where three forces balance: **weight of float (down) = buoyancy (up) + fluid drag (up)**. A bigger flow needs a bigger gap, so the float rides higher.
5. Unlike the orifice meter, the pressure drop across the float stays nearly *constant*; the *area* changes instead — hence "variable-area meter".

How the reading is taken. Read the scale at the float's reference edge — usually the *top* of the float (follow the manufacturer's mark; many floats have a reading line). Read at eye level to avoid parallax error. The scale may read directly in L/min or as a percentage of maximum.

Calibration concept. The scale is only trustworthy if the meter was calibrated: the manufacturer (or plant engineer) passes *known* flow rates through the meter and marks where the float sits for each. Because float position depends on fluid density and viscosity, a rotameter calibrated for water will misread a syrup — it must be recalibrated (or a correction factor applied) for a different fluid.

> *Worked numerical (k).* A rotameter is calibrated: 50 L/min at float height 25 cm. Estimate the float position for 35 L/min, assuming a linear scale through the origin.

- **Assumption stated clearly:** real rotameter scales are only *approximately* linear; this treats the scale as a straight line through zero as an approximation.
- Position = (35 / 50) × 25 cm = 0.7 × 25 = 17.5 cm.

Answer: ≈ 17.5 cm (approximate — linear-scale assumption).

> **Key takeaway:** The rotameter is the simplest direct-reading meter — no manometer, no calculation — but its scale belongs to one fluid only.

4.5 Classifications

Classification of flow meters (by working principle):

- **Differential-pressure (head) meters** — fixed constriction, ΔP varies: orifice meter, venturimeter, pitot tube (velocity from ΔP).
- **Variable-area meters** — ΔP nearly constant, area varies: rotameter.
- (Beyond syllabus: electromagnetic, turbine, ultrasonic meters — mentioned only so the student knows the family is bigger.)

Classification by what they measure:

- Volumetric flow rate Q: orifice meter, venturimeter, rotameter.
- Local velocity v: pitot tube (Q derived after averaging).

4.6 Mechanisms (step-by-step)

Mechanism 1: Orifice meter — from pressure drop to flow rate.

1. Fluid hits the orifice plate; only the hole lets it through.
2. Jet contracts to the vena contracta → velocity peaks, pressure dips.
3. Downstream the jet expands; turbulence dissipates energy (permanent loss).
4. Manometer reads Δh between upstream and downstream tappings.
5. Convert h to flowing-fluid head: $\Delta h = h(\rho_m/\rho_f - 1)$.
6. Apply $Q = Co \cdot Ao \cdot \sqrt{(2g\Delta h)}/\sqrt{(1-\beta^4)}$; Co corrects for friction and the vena contracta.

Mechanism 2: Venturimeter — smooth acceleration and pressure recovery.

1. Convergent cone accelerates fluid smoothly to the throat.
2. Throat: highest velocity, lowest pressure — manometer tapping reads P2.
3. Inlet tapping reads P1; $\Delta h = P1 - P2$ as head.
4. Divergent cone decelerates fluid gradually, recovering most pressure (small permanent loss).
5. Compute $Q = Cd \cdot A1 \cdot A2 \cdot \sqrt{(2g\Delta h)}/\sqrt{(A1^2 - A2^2)}$; Cd ≈ 0.97.

Mechanism 3: Pitot tube — stopping the fluid to weigh its speed.

1. Forward opening faces the flow; fluid stagnates → pressure rises to stagnation value.
2. Side opening senses undisturbed static pressure.
3. Manometer difference = dynamic head $\Delta h = v^2/2g$.
4. Invert: $v = \sqrt{(2g\Delta h)}$ gives the local velocity at the tip.

Mechanism 4: Rotameter — the float finds its level.

1. Flow starts upward; drag lifts the float off its stop.
2. Float rises into the wider tube section; the gap around it grows.
3. Larger gap → lower velocity past the float → less drag.
4. Float stops where weight = buoyancy + drag (force balance).
5. Read the scale at the float's top edge; the marked value is the flow rate (for the calibrated fluid).

4.7 Diagrams (textual, labelled — draw from these descriptions)

> **Diagram 4.1 — Orifice meter.** Draw a horizontal pipe with a vertical plate across it in the middle. Label: (1) pipe, diameter D; (2) orifice plate (thin vertical line); (3) orifice hole, diameter d, sharp edges; (4) upstream pressure tapping (small port just before the plate); (5) downstream tapping (just after the plate, near the vena contracta); (6) U-tube differential manometer joining the two tappings, showing level difference h; (7) vena contracta — dashed jet outline narrowing just after the plate, labelled "narrowest jet, lowest pressure". Title: "Orifice meter — sharp plate creates a measured pressure drop".

> **Diagram 4.2 — Venturimeter.** Draw a horizontal pipe that narrows smoothly then widens. Label: (1) inlet pipe, diameter D1, tapping P1; (2) convergent cone (gentle narrowing); (3) throat, diameter D2, tapping P2 (lowest pressure); (4) divergent cone (long gentle widening); (5) outlet pipe back to D1; (6) differential manometer between tappings 1 and 3 showing Δh; (7) arrow of flow left to right. Title: "Venturimeter — smooth constriction, small permanent pressure loss".

Diagram 4.3 — Pitot tube. Draw a horizontal pipe with flow arrows. Inside, an L-shaped tube: vertical leg rising out of the pipe, bent so its open tip faces left (into the flow). Label: (1) forward-facing opening (stagnation point — fluid stops dead); (2) side openings on the outer sleeve (static pressure); (3) inner tube to manometer (stagnation pressure); (4) outer tube to manometer (static pressure); (5) differential manometer showing Δh; (6) streamline sketch bending into the tip. Title: "Pitot tube — stagnation pressure minus static pressure gives velocity".

Diagram 4.4 — Rotameter. Draw a vertical tapered tube (narrow bottom, wide top) with a scale on the side. Label: (1) fluid inlet at bottom; (2) tapered metering tube; (3) float (spinning bob) shown at mid-height; (4) annular gap between float and tube wall (label "gap grows as float rises"); (5) scale with marks (e.g., 10, 20, 30 L/min); (6) reading line at the top edge of the float; (7) fluid outlet at top; (8) force arrows on float: weight down, buoyancy + drag up. Title: "Rotameter — float height reads flow rate directly".

4.8 Flowcharts

Flowchart 4.1 — Choosing a flow meter for a given fluid and duty.

```
START: what are you measuring?
  |
  +-- Need LOCAL VELOCITY at a point (e.g., air duct traverse)?
  |       --> Pitot tube (cheap, simple; average several points for Q)
  |
  +-- Need FLOW RATE Q in a pipe?
  |       |
  |       +-- Clean liquid, want LOW COST and can accept pressure loss?
  |       |       --> Orifice meter
  |       |
  |       +-- Want ACCURACY + LOW permanent loss (pump energy matters)?
  |       |       --> Venturimeter (costs more, needs more space)
  |       |
  |       +-- Slurry / viscous / solids present?
  |               --> Venturimeter (no sharp edge to clog or erode)
  |
  +-- Need DIRECT visual reading, small line, one fluid?
          --> Rotameter (check calibration fluid matches!)
```

Flowchart 4.2 — From manometer reading to flow rate (orifice/venturi).

```
START: read manometer height h
  |
  v
Convert to flowing-fluid head: Dh = h * (rho_m/rho_f - 1)
  |
  v
Pick formula: orifice -> Q = Co*Ao*sqrt(2g*Dh)/sqrt(1-b^4)
              venturi -> Q = Cd*A1*A2*sqrt(2g*Dh)/sqrt(A1^2-A2^2)
  |
  v
Insert Co or Cd (given / calibrated)
  |
  v
Compute Q (m3/s); multiply by 1000 for L/s
```

4.9 Infographic concept box

```
┌─ INFOGRAPHIC: THE FOUR FLOW METERS ──────────────────────┐
│ Draw four panels in a row: │
│ │
│ ① ORIFICE — plate with a hole. "Cheap & simple. │
│ Sharp edge wastes energy (Co ≈ 0.62)." │
│ ② VENTURI — smooth hourglass. "Accurate & gentle. │
│ Recovers pressure (Cd ≈ 0.97)." │
│ ③ PITOT — L-tube facing the flow. "Stops fluid dead; │
│ pressure of stopping = speed. v = √(2gΔh)." │
│ ④ ROTAMETER — float in a tapered tube. "Float height │
│ = flow rate. Read the scale; mind the calibration fluid." │
│ │
│ Footer: "1-3: measure ΔP (Bernoulli). 4: measures area (float)." │
└──────────────────────────────────────────────────────────┘
```

4.10 Tables

Table 4.1 — The four flow meters compared (full comparison).

Feature	Orifice meter	Venturimeter	Pitot tube	Rotameter
Principle	ΔP across sharp orifice (Bernoulli)	ΔP across smooth throat (Bernoulli)	Stagnation vs static pressure	Variable area; float force balance
Measures	Flow rate Q	Flow rate Q	Local velocity v	Flow rate Q (direct reading)
Formula	$Q = Co \cdot Ao\sqrt{(2g\Delta h)/\sqrt{(1-\beta^4)}}$	$Q = Cd \cdot A1A2\sqrt{(2g\Delta h)/\sqrt{(A1^2-A2^2)}}$	$v = \sqrt{(2g\Delta h)}$	Scale reading (calibrated)
Discharge coefficient	Co ≈ 0.6–0.65	Cd ≈ 0.95–0.98	≈ 1.0 (probe factor)	Not applicable (scale)
Permanent pressure loss	High	Low	Very low (tiny probe)	Moderate, nearly constant
Cost	Low	High	Very low	Low–moderate
Accuracy	Moderate	High	Moderate (point velocity)	Moderate (for calibrated fluid)
Space needed	Short	Long	Minimal	Vertical straight run
Viscous/slurry fluids	Poor (clogs, erodes edge)	Good	Fair	Fair (recalibrate)
Extra instrument	Differential manometer	Differential manometer	Differential manometer	None (built-in scale)
Typical pharma use	Utility water lines	Precise dosing lines	Air-duct velocity checks	Purge gas, small liquid lines

Table 4.2 — Orifice meter vs venturimeter (head-to-head).

Point	Orifice meter	Venturimeter
Construction	Flat plate with sharp hole	Convergent cone + throat + divergent cone
Cd/Co	Co ≈ 0.62 (more friction)	Cd ≈ 0.97 (smooth, near-ideal)
Pressure recovery	Poor — energy lost as turbulence	Good — divergent cone recovers pressure
Cost & size	Cheap, compact	Expensive, long
Wear	Sharp edge erodes; bad for slurries	No sharp edge; handles slurries
Best when	Budget matters, loss acceptable	Accuracy and energy saving matter

4.11 Examples

- **Purified-water loop:** An orifice meter with a differential pressure transmitter monitors circulation flow in a utility water line — cheap and good enough where a little pressure loss does not matter.
- **Syrup dosing:** A venturimeter on a viscous syrup transfer line gives accurate flow readings with minimal pressure loss, so the dosing pump works less hard and shear on the product stays low.
- **HVAC duct check:** A technician inserts a pitot tube into an air-handling duct, takes velocity readings at several points, averages them, and confirms the cleanroom gets its designed air changes.
- **Nitrogen purging:** A small rotameter on a nitrogen cylinder line lets the operator set and see the purge flow (e.g., 10 L/min) directly while blanketing a solvent storage tank.
- **Filter watch:** The same differential-manometer idea from Chapter 3 appears on the tappings of these meters — one instrument family, many jobs.

4.12 Analogies

Orifice plate = toll plaza speed-breaker. Cars (fluid) must squeeze through a few narrow booths; the queue (pressure) builds up before the booths and the road is freer after. Counting the queue tells you the traffic flow.

Venturimeter = smooth flyover. Instead of a jolting speed-breaker, traffic glides up a smooth ramp, speeds through a narrow stretch, and glides down — less fuel wasted, same measurement. That is why its Cd is near 1.

Pitot tube = putting your palm out of a moving auto. Your palm facing the wind feels the full push (stagnation); the back of your hand feels less. The difference tells you how fast you are going.

Rotameter = a beach ball in a waterfall pipe. Drop a beach ball into an upward-flowing pipe that widens toward the top: stronger flow pushes the ball higher until the wider gap lets water slip past easily. The ball's height is the flow reading.

Discharge coefficient = marks lost to silly mistakes. The ideal formula is the "full marks" answer; friction and turbulence are the "silly mistakes" that cost marks — Co/Cd is the fraction of marks you actually keep.

4.13 Pharmacy relevance

- **Water systems:** Purified-water and water-for-injection loops are the lifelines of a formulation plant; orifice and venturi meters track circulation and distribution flows continuously.
- **Accurate dosing:** Syrup, suspension and liquid-injection filling lines need precise flow measurement — venturimeters give the accuracy that keeps fill volumes within specification.
- **Cleanroom HVAC:** Pitot-tube traverses verify air velocities in ducts serving sterile areas, supporting environmental qualification.
- **Inert blanketing:** Rotameters on nitrogen lines give operators a direct visual setting for tank blanketing and purging during solvent handling.
- **Energy saving:** Choosing a venturimeter over an orifice plate where pump energy is significant reduces permanent pressure loss — a real operating-cost decision.
- **Shear-sensitive products:** Low-loss, smooth-contour meters (venturi) disturb emulsions and protein solutions less than sharp-edged plates.

4.14 India context

- Indian pharmaceutical plants commonly use orifice plates paired with differential pressure transmitters on utility lines (purified water, steam condensate) because they are inexpensive, standardised and easy to replace during maintenance shutdowns.
- Venturimeters are specified on critical process lines — for example, accurate transfer of bulk syrup or solvent — where the higher upfront cost is recovered through lower pumping energy and better accuracy.
- Pitot tubes remain a standard tool for HVAC validation contractors who qualify cleanroom air-handling systems in Indian sterile facilities; the multi-point traverse method is part of routine duct testing practice.
- Glass-tube rotameters are a familiar sight on gas lines (nitrogen, compressed air) in Indian formulation and API units, giving operators an immediate visual flow reading without any electronics.
- As with manometers, the industry trend is toward digital/electronic versions of these meters, but the Bernoulli-based working principles taught here are exactly what the electronic instruments implement.

4.15 Key points

- Orifice, venturi and pitot share one principle: constriction → faster flow → lower pressure (Bernoulli + continuity); ΔP reveals the flow.
- Orifice meter: $Q = Co \cdot Ao \cdot \sqrt{(2g\Delta h)}/\sqrt{(1-\beta^4)}$; $\beta = d/D$; $Co \approx 0.62$; cheap but high permanent pressure loss.
- Venturimeter: $Q = Cd \cdot A1 \cdot A2 \cdot \sqrt{(2g\Delta h)}/\sqrt{(A1^2 - A2^2)}$; $Cd \approx 0.97$; smooth convergent–throat–divergent shape; low loss, high accuracy, handles slurries.
- Venturi advantages over orifice: lower permanent loss, higher Cd, less wear/clogging, better for viscous fluids; drawback: bigger, costlier.
- Pitot tube: $v = \sqrt{(2g\Delta h)}$ from stagnation minus static pressure; measures point velocity — average several points for Q.
- Rotameter: variable-area meter; float rises until weight = buoyancy + drag; read the scale at the float's top edge; calibrated for one fluid — recalibrate for another.
- $\Delta h = h(\rho_m/\rho_f - 1)$ converts a manometer reading to head of the flowing fluid.

4.16 Common confusions

- **Cd vs Co:** Both are discharge coefficients (< 1). Co belongs to the orifice meter (~0.62); Cd belongs to the venturimeter (~0.97). Do not swap them.
- **Δh vs h:** h is the manometer column height; Δh is that height converted to *head of the flowing fluid* via (ρ_m/ρ_f − 1). Formulas need Δh, not raw h.
- **Pitot measures v, not Q:** The pitot tube gives velocity at one point. Students often plug it straight into Q = Av with the centre velocity — average across the section first.
- **Rotameter ΔP:** In a rotameter the pressure drop stays nearly constant while the *area* changes — the opposite of orifice/venturi meters. Do not apply ΔP formulas to it.
- **Vena contracta vs throat:** The vena contracta is the jet's natural pinch *after* an orifice plate; the throat is the *built* narrow section of a venturimeter. Different devices, different terms.
- **β definition:** β = d/D (orifice/pipe), a ratio less than 1. β^4 in the orifice formula is often small but not negligible.

4.17 Common mistakes

- Using h directly in place of Δh = h(ρ_m/ρ_f − 1) — the classic 12.6× error with mercury/water.
- Forgetting β^4 under the square root in the orifice formula, or using β^2 instead of β^4.
- Mixing up A1 and A2 in the venturi formula (A1 = pipe/inlet, A2 = throat).
- Reading the rotameter at the bottom of the float instead of the top edge/reading line.
- Using a water-calibrated rotameter for a syrup or oil without correction — the reading will be wrong.
- Taking a single centre-line pitot reading as the average velocity — centre velocity is the maximum, so Q will be overestimated.
- Applying Bernoulli between points across a pump, valve or the orifice plate itself (friction/loss zones) without a loss term.

4.18 Mnemonics

- **Four meters — "OVPR":** *"Orifice, Venturi, Pitot, Rotameter — Old Villages Pump Rapidly"* — in the order: cheap plate, smooth tube, L-probe, float.
- **Venturi beats orifice — "CALM":** *"Cd high, Accurate, Low loss, Handles slurries, More costly"* — why venturi wins (except price).
- **Rotameter reading — "Top of the float, eye to the dot":** read at the float's top edge, at eye level.
- **Pitot — "Stop to clock speed":** the tube *stops* the fluid; the stopping pressure *clocks* the speed — v = √(2gΔh).

4.19 Exam summary

The orifice meter, venturimeter and pitot tube all work on Bernoulli's principle plus continuity: a constriction raises velocity and lowers pressure, and the measured ΔP gives the flow. Orifice meter: thin sharp-edged plate, Q = Co·Ao√(2gΔh)/√(1−β^4), Co ≈ 0.62 — cheap but lossy. Venturimeter: smooth convergent–throat–divergent section, Q = Cd·A1A2√(2gΔh)/√($A1^2$−$A2^2$), Cd ≈ 0.97 — accurate, low permanent loss, better for slurries and viscous fluids, but larger and costlier. Pitot tube: L-shaped probe; stagnation pressure minus static pressure gives point velocity v = √(2gΔh); average several points for Q. Rotameter: variable-area meter with a float in a tapered vertical tube; float height (read at its top edge) directly indicates flow rate; the scale is calibrated for one fluid, so recalibration is needed for another. Δh = h(ρ_m/ρ_f − 1) converts any manometer reading to flowing-fluid head.

4.20 One-minute revision

- 3 meters use ΔP (Bernoulli): orifice, venturi, pitot. Rotameter uses float height.
- Orifice: Co ≈ 0.62, cheap, high loss. Venturi: Cd ≈ 0.97, accurate, low loss.
- Pitot: v = √(2gΔh); point velocity only.
- Rotameter: read top of float; calibrated per fluid.
- Always convert h → Δh = h(ρ_m/ρ_f − 1).
- Venturi advantages: low permanent loss, high Cd, no sharp edge, slurry-friendly.

4.21 Exam questions

Long/essay questions (5):

1. Describe the construction and working of a venturimeter with a neat labelled diagram. Derive its discharge formula and list its advantages over the orifice meter.
2. Explain the construction and working of an orifice meter. Derive the formula $Q = Co \cdot Ao \cdot \sqrt{(2g\Delta h)}/\sqrt{(1-\beta^4)}$ and explain the meaning of Co and β.
3. Describe the pitot tube with a labelled diagram. How does it measure velocity? Why must several readings be averaged to find the flow rate?
4. Explain the working principle of a rotameter with a labelled diagram. How is the reading taken, and why must the meter be calibrated for the fluid in use?
5. Compare the orifice meter, venturimeter, pitot tube and rotameter under the heads: principle, formula, pressure loss, cost, accuracy and suitability for viscous fluids.

Short questions (8):

1. State the common working principle of the orifice meter, venturimeter and pitot tube.
2. What is the vena contracta?
3. Define coefficient of discharge. Why is it less than 1?
4. Write the venturimeter discharge formula and name each symbol.
5. Give three advantages of the venturimeter over the orifice meter.
6. What is stagnation pressure?
7. How is a rotameter reading taken? Where exactly do you read the scale?
8. Why does a rotameter need recalibration when the fluid is changed?

MCQs (10) — answer key at the end:

1. The coefficient of discharge of a venturimeter is typically about —
   (a) 0.62 (b) 0.75 (c) 0.97 (d) 1.20
2. The coefficient of discharge of an orifice meter is typically about —
   (a) 0.97 (b) 0.62 (c) 1.00 (d) 0.85
3. In an orifice meter, β is defined as —
   (a) D/d (b) d/D (c) D^2/d^2 (d) d^2/D^2
4. The pitot tube measures —
   (a) flow rate directly (b) pressure difference (c) point velocity (d) viscosity
5. The velocity measured by a pitot tube is given by —
   (a) √(gh) (b) √(2gΔh) (c) 2gΔh (d) gΔh/2
6. A rotameter is a —
   (a) differential-pressure meter (b) variable-area meter (c) velocity probe (d) mass flow meter
7. In a rotameter, the float comes to rest when —
   (a) weight = drag only (b) weight = buoyancy + drag (c) buoyancy = drag only (d) flow stops
8. Which meter has the lowest permanent pressure loss?
   (a) Orifice meter (b) Venturimeter (c) Pitot tube inserted in a large duct (d) Rotameter
9. The manometer reading h is converted to flowing-fluid head Δh by —
   (a) h(ρ_m + ρ_f) (b) h(ρ_m/ρ_f − 1) (c) h(ρ_f/ρ_m) (d) h·ρ_m·g
10. Which device is most suitable for a slurry that would erode a sharp edge?
    (a) Orifice meter (b) Venturimeter (c) Pitot tube (d) Inclined manometer

Answer key: 1-(c), 2-(b), 3-(b), 4-(c), 5-(b), 6-(b), 7-(b), 8-(c), 9-(b), 10-(b).

Viva questions (5) with brief answers:

1. *Why is the venturimeter's Cd (≈0.97) so much higher than the orifice meter's Co (≈0.62)?* — The venturi's smooth convergent/divergent shape avoids the sharp-edge turbulence and vena-contracta losses of the orifice plate, so real flow is close to ideal.
2. *What is the vena contracta and where does it occur?* — The narrowest cross-section of the jet just downstream of an orifice plate, where velocity is maximum and pressure minimum.
3. *Can you use a pitot tube to get flow rate in one reading?* — No — it gives velocity at one point only; you traverse several points across the section, average them, then multiply by area.
4. *Why must a rotameter be mounted vertically?* — The float's force balance (weight vs buoyancy + drag) only works with flow rising vertically through the tapered tube; tilting changes the balance and the reading.
5. *Your mercury manometer on a venturimeter reads h = 0.20 m for water flow. What is Δh in metres of water?* — Δh = 0.20 × (13 600/1 000 − 1) = 0.20 × 12.6 = 2.52 m of water.

Application-based questions (3):

1. A plant wants to meter purified water on a utility line where budget is tight and a moderate pressure drop is acceptable. A second line doses an expensive viscous syrup where accuracy and low shear matter. Recommend a meter for each line and justify.
2. A pitot-tube traverse of a cleanroom supply duct gives centre velocity 4.0 m/s and an average of 3.6 m/s over the section (area 0.25 m^2). Calculate the air flow rate in m^3/s. What error would result from using the centre reading alone?
3. An operator moves a water-calibrated rotameter to a glycerine service line and trusts the scale. Explain, using the float force balance, why the reading will mislead and what should be done instead.

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